International Baccalaureate®
Baccalauréat International
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MATHEMATICS
STANDARD LEVEL
PAPER 2

Thursday 8 May 2008 (morning)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 7]*

In a school with 125 girls, each student is tested to see how many sit-up exercises (sit-ups) she can do in one minute. The results are given in the table below.

Number of sit-ups	Number of students	Cumulative number of students
15	11	11
16	21	32
17	33	p
18	q	99
19	18	117
20	8	125

(This question continues on the following page)

(Question 1 continued)

(a) (i) Write down the value of p.

(ii) Find the value of q. *[3 marks]*

(b) Find the median number of sit-ups. *[2 marks]*

(c) Find the mean number of sit-ups. *[2 marks]*

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2. *[Maximum mark: 6]*

The diagram below shows triangle PQR. The length of [PQ] is 7 cm, the length of [PR] is 10 cm, and $\hat{PQR}$ is 75°.

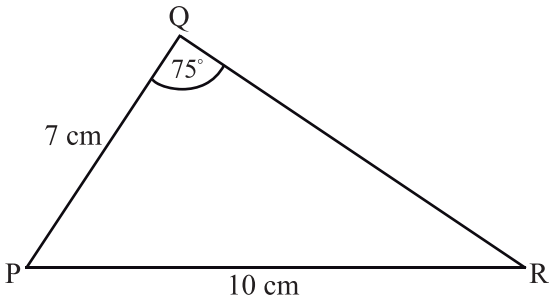

diagram not to scale

(a) Find $\hat{PRQ}$. *[3 marks]*

(b) Find the area of triangle PQR. *[3 marks]*

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3. *[Maximum mark: 6]*

The diagram below shows a circle centre O, with radius *r*. The length of arc ABC is 3π cm and $\text{A}\hat{\text{O}}\text{C} = \frac{2\pi}{9}$.

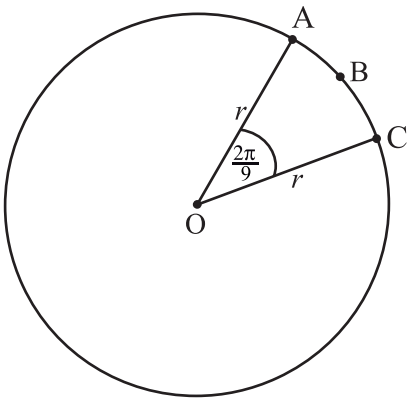

diagram not to scale

(a) Find the value of *r*. *[2 marks]*

(b) Find the perimeter of sector OABC. *[2 marks]*

(c) Find the area of sector OABC. *[2 marks]*

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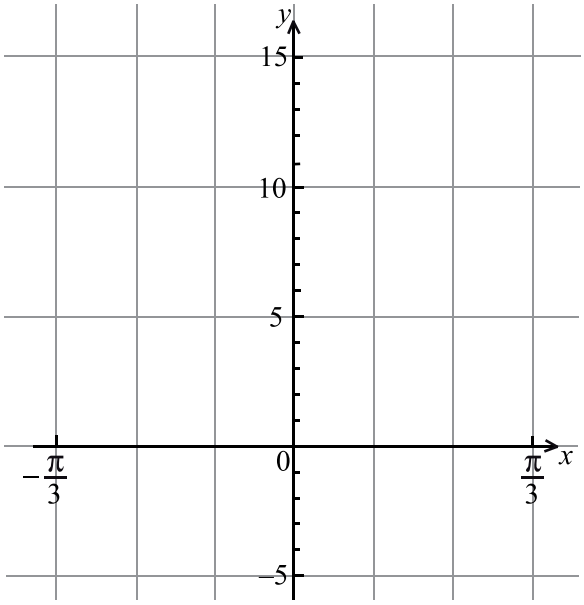
4. *[Maximum mark: 6]*

Let $f(x) = 4\tan^2 x - 4\sin x$, $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$.

(a) On the grid below, sketch the graph of $y = f(x)$. *[3 marks]*



(b) Solve the equation $f(x) = 1$. *[3 marks]*

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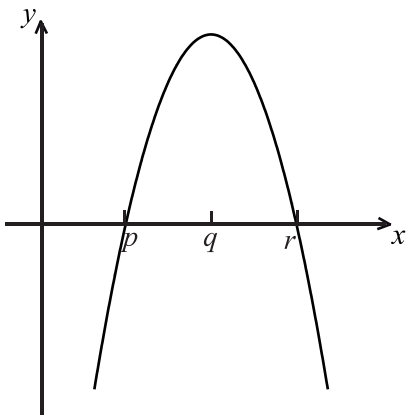
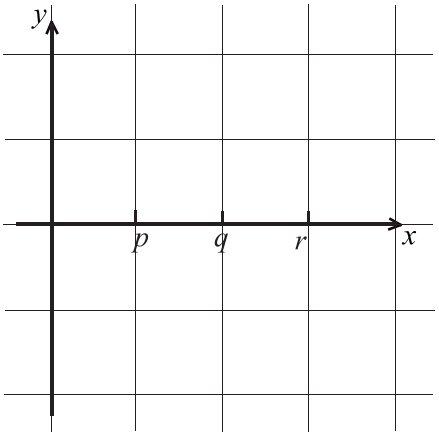
5. *[Maximum mark: 6]*

The diagram below shows part of the graph of the **gradient** function, $y = f'(x)$.



(a) On the grid below, sketch a graph of $y = f''(x)$, clearly indicating the *x*-intercept. *[2 marks]*



(b) Complete the table, for the graph of $y = f(x)$. *[2 marks]*

	x-coordinate
(i) Maximum point on f	
(ii) Inflexion point on f	

(c) Justify your answer to part (b)(ii). *[2 marks]*

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6. *[Maximum mark: 7]*

A factory makes switches. The probability that a switch is defective is 0.04. The factory tests a random sample of 100 switches.

(a) Find the mean number of defective switches in the sample. *[2 marks]*

(b) Find the probability that there are exactly six defective switches in the sample. *[2 marks]*

(c) Find the probability that there is at least one defective switch in the sample. *[3 marks]*

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7. *[Maximum mark: 7]*

Let $\boldsymbol{v} = 3\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{w} = \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$. The vector $\boldsymbol{v} + p\boldsymbol{w}$ is perpendicular to $\boldsymbol{w}$. Find the value of p.

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 13]*

A box contains a large number of biscuits. The weights of biscuits are normally distributed with mean 7 g and standard deviation 0.5 g.

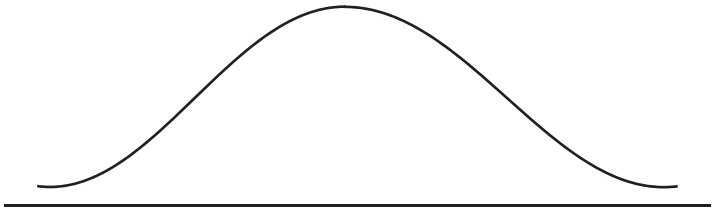
(a) One biscuit is chosen at random from the box. Find the probability that this biscuit

(i) weighs less than 8 g;

(ii) weighs between 6 g and 8 g. *[4 marks]*

(b) Five percent of the biscuits in the box weigh less than d grams.

(i) Copy and complete the following normal distribution diagram, to represent this information, by indicating d, and shading the appropriate region.



(ii) Find the value of d. *[5 marks]*

(c) The weights of biscuits in another box are normally distributed with mean μ and standard deviation 0.5 g. It is known that 20 % of the biscuits in this second box weigh less than 5 g.

Find the value of μ. *[4 marks]*

9. *[Maximum mark: 18]*

The point O has coordinates $(0, 0, 0)$, point A has coordinates $(1, -2, 3)$ and point B has coordinates $(-3, 4, 2)$.

(a) (i) Show that $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

(ii) Find $B\hat{A}O$. *[8 marks]*

(b) The line L_1 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s\begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

Write down the coordinates of two points on L_1. *[2 marks]*

(c) The line L_2 passes through A and is parallel to $\overrightarrow{OB}$.

(i) Find a vector equation for L_2, giving your answer in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

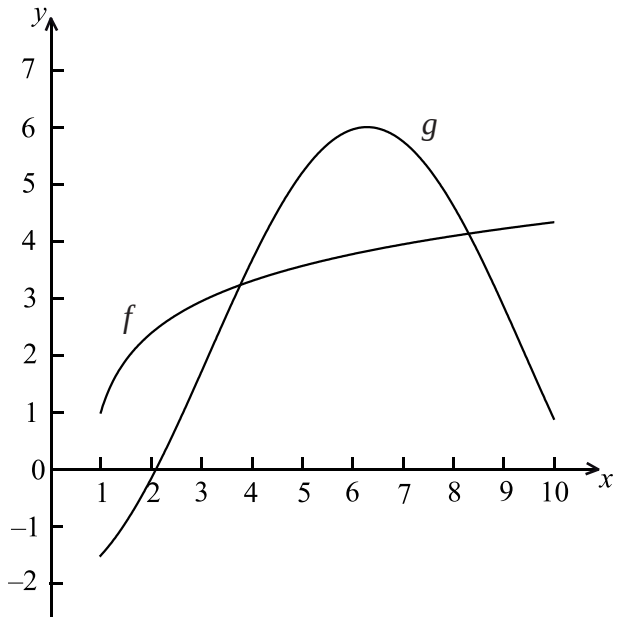
(ii) Point $C(k, -k, 5)$ is on L_2. Find the coordinates of C. *[6 marks]*

(d) The line L_3 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, and passes through the point C.

Find the value of p at C. *[2 marks]*

10. *[Maximum mark: 14]*

The following diagram shows the graphs of $f(x) = \ln(3x-2)+1$ and $g(x) = -4\cos(0.5x)+2$, for $1 \le x \le 10$.



(a) Let A be the area of the region **enclosed** by the curves of f and g.

(i) Find an expression for A.

(ii) Calculate the value of A. *[6 marks]*

(b) (i) Find $f'(x)$.

(ii) Find $g'(x)$. *[4 marks]*

(c) There are two values of x for which the gradient of f is equal to the gradient of g. Find both these values of x. *[4 marks]*